Yr 11 -12
Transition Document

Research Task 1

Simplifying algebraic fractions

A LEVEL LINKS

Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Example 1 Factorise $x^2 + 3x - 10$

$b = 3$, $ac = -10$ So $x^2 + 3x - 10 = x^2 + 5x - 2x - 10$ $= x(x + 5) - 2(x + 5)$ $= (x + 5)(x - 2)$	**1** Work out the two factors of $ac = -10$ which add to give $b = 3$ (5 and -2) **2** Rewrite the b term ($3x$) using these two factors **3** Factorise the first two terms and the last two terms **4** $(x + 5)$ is a factor of both terms

Example 2 Factorise $6x^2 - 11x - 10$

$b = -11$, $ac = -60$ So $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$ $= 3x(2x - 5) + 2(2x - 5)$ $= (2x - 5)(3x + 2)$	**1** Work out the two factors of $ac = -60$ which add to give $b = -11$ (-15 and 4) **2** Rewrite the b term ($-11x$) using these two factors **3** Factorise the first two terms and the last two terms **4** $(2x - 5)$ is a factor of both terms

Example 3 Factorise $4x^2 - 25y^2$

$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$	This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$

Example 4 Simplify $\dfrac{x^2 - 4x - 21}{2x^2 + 9x + 9}$

$\dfrac{x^2 - 4x - 21}{2x^2 + 9x + 9}$	**1** Factorise the numerator and the denominator
For the numerator: $b = -4$, $ac = -21$	**2** Work out the two factors of $ac = -21$ which add to give $b = -4$ (-7 and 3)
So $x^2 - 4x - 21 = x^2 - 7x + 3x - 21$	**3** Rewrite the b term ($-4x$) using these two factors
$= x(x - 7) + 3(x - 7)$	**4** Factorise the first two terms and the last two terms
$= (x - 7)(x + 3)$	**5** $(x - 7)$ is a factor of both terms
For the denominator: $b = 9$, $ac = 18$	**6** Work out the two factors of $ac = 18$ which add to give $b = 9$ (6 and 3)
So $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$	**7** Rewrite the b term ($9x$) using these two factors
$= 2x(x + 3) + 3(x + 3)$	**8** Factorise the first two terms and the last two terms
$= (x + 3)(2x + 3)$	**9** $(x + 3)$ is a factor of both terms
So $\dfrac{x^2 - 4x - 21}{2x^2 + 9x + 9} = \dfrac{(x - 7)(x + 3)}{(x + 3)(2x + 3)}$	**10** $(x + 3)$ is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1
$= \dfrac{x - 7}{2x + 3}$	

Video Link

Below is a video link to a large collection of math tutorials. Every chapter we will cover in the course has a video.

To help with the questions on the next slide have a look at the chapter 7 material, video 35, algebraic fractions.

- Introduction - YouTube

Practice questions

1 Simplify the algebraic fractions.

a $\dfrac{2x^2+4x}{x^2-x}$

b $\dfrac{x^2+3x}{x^2+2x-3}$

c $\dfrac{x^2-2x-8}{x^2-4x}$

d $\dfrac{x^2-5x}{x^2-25}$

e $\dfrac{x^2-x-12}{x^2-4x}$

f $\dfrac{2x^2+14x}{2x^2+4x-70}$

2 Simplify

a $\dfrac{9x^2-16}{3x^2+17x-28}$

b $\dfrac{2x^2-7x-15}{3x^2-17x+10}$

c $\dfrac{4-25x^2}{10x^2-11x-6}$

d $\dfrac{6x^2-x-1}{2x^2+7x-4}$

Extend

3 Simplify $\sqrt{x^2+10x+25}$

4 Simplify $\dfrac{(x+2)^2+3(x+2)^2}{x^2-4}$

Answers on the next pages, do make sure you attempt all questions before looking at the answers.

If you are stuck on any question, even after marking, refer to the video link, search online for any other help you can find. If you are still stuck, you can always ask a teacher or anyone else for help.

Answers – Practice Questions

Answers

1 **a** $\frac{2(x+2)}{x-1}$ **b** $\frac{x}{x-1}$

c $\frac{x+2}{x}$ **d** $\frac{x}{x+5}$

e $\frac{x+3}{x}$ **f** $\frac{x}{x-5}$

2 **a** $\frac{3x+4}{x+7}$ **b** $\frac{2x+3}{3x-2}$

c $\frac{2-5x}{2x-3}$ **d** $\frac{3x+1}{x+4}$

Answers - Extend

3 $(x + 5)$

4 $\dfrac{4(x+2)}{x-2}$

Research Task 2

Pearson
Edexcel

Angles in all four quadrants

A LEVEL LINKS
Scheme of work: 4a. Trigonometric ratios and graphs

Key points

- The sine, cosine and tangent of some angles may be written exactly.

	0	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	

- You can use these rules to find sin, cos and tan of any positive or negative angle using the corresponding cute angle made with the x-axis, θ.

$\sin(180° - \theta) = \sin\theta$
$\cos(180° - \theta) = -\cos\theta$
$\tan(180° - \theta) = -\tan\theta$
$\sin(180° + \theta) = -\sin\theta$
$\cos(180° + \theta) = -\cos\theta$
$\tan(180° + \theta) = \tan\theta$

Video Link

Same link as before.

This time look at chapter 10.

Starting with video 54.

- Introduction - YouTube

Practice questions

1 Without using a calculator, write down the values of:

(a) $\sin 90°$ (b) $\cos 270°$ (c) $\tan 360°$ (d) $\sin 270°$

2 Express the following in terms of trigonometric ratios of acute angles:

(a) $\sin(-200°)$ b) $\cos(-200°)$ (c) $\tan(-40°)$ (d) $\tan 335°$

3 Express the following as trigonometric ratios of either 30°, 45° or 60°, and hence find their exact values:

(a) $\sin(-135°)$ (b) $\cos 225°$ (c) $\tan(-120°)$ (d) $\sin(-200°)$

Extend

4.

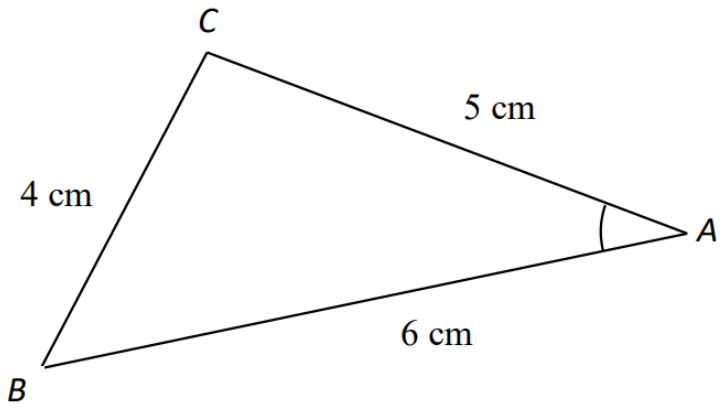


Figure 1

Figure 1 shows the triangle ABC, with $AB = 6$ cm, $BC = 4$ cm and $CA = 5$ cm.

(a) Show that $\cos A = \frac{3}{4}$.

(b) Hence, or otherwise, find the exact value of $\sin A$.

Answers – Practice Questions

Answers

1	(a)	1	(b)	0	(c)	0	(d)	-1
2	(a)	$\sin 20°$	(b)	$-\cos 20°$	(c)	$-\tan 40°$	(d)	$-\tan 25°$
3	(a)	$-\frac{1}{\sqrt{2}}$	(b)	$-\frac{1}{\sqrt{2}}$	(c)	$\sqrt{3}$	(d)	$--\frac{\sqrt{3}}{2}$

Answers - Extend

4 (a) $4^2 = 5^2 + 6^2 - (2 \times 5 \times 6 \cos\theta)$

$$\cos\theta = \frac{5^2 + 6^2 - 4^2}{2 \times 5 \times 6}$$

$$\left(= \frac{45}{60}\right) = \frac{3}{4}$$

(b) $\sin^2 A + \left(\frac{3}{4}\right)^2 = 1$

$$\left(\sin^2 A = \frac{7}{16}\right) \quad \sin A = \frac{1}{4}\sqrt{7}$$

Exam Questions

This section contains exam questions relating to the two research topics.

Exam Questions 1

2 Simplify these fractions as far as possible:

a $\dfrac{(x+3)(x-2)}{(x-2)}$ **b** $\dfrac{(x+4)(3x-1)}{(3x-1)}$ **c** $\dfrac{(x+3)^2}{(x+3)}$

d $\dfrac{x^2+10x+21}{(x+3)}$ **e** $\dfrac{x^2+9x+20}{(x+4)}$ **f** $\dfrac{x^2+x-12}{(x-3)}$

g $\dfrac{x^2+x-20}{x^2+2x-15}$ **h** $\dfrac{x^2+3x+2}{x^2+5x+4}$ **i** $\dfrac{x^2+x-12}{x^2-9x+18}$

E/P **3** $\dfrac{6x^3+3x^2-84x}{6x^2-33x+42} = \dfrac{ax(x+b)}{x+c}$, where a, b and c are constants.

Work out the values of a, b and c. **(4 marks)**

Answers Exam Set 1

2 a $\dfrac{(x+3)(x-2)}{(x-2)} = x+3$

b $\dfrac{(x+4)(3x-1)}{(3x-1)} = x+4$

c $\dfrac{(x+3)^2}{(x+3)} = \dfrac{(x+3)(x+3)}{(x+3)}$

$= x+3$

d $\dfrac{x^2+10x+21}{(x+3)} = \dfrac{(x+7)(x+3)}{(x+3)}$

$= x+7$

e $\dfrac{x^2+9x+20}{(x+4)} = \dfrac{(x+4)(x+5)}{(x+4)}$

$= x+5$

f $\dfrac{x^2+x-12}{(x-3)} = \dfrac{(x-3)(x+4)}{(x-3)}$

$= x+4$

g $\dfrac{x^2+x-20}{x^2+2x-15} = \dfrac{(x+5)(x-4)}{(x+5)(x-3)}$

$= \dfrac{x-4}{x-3}$

h $\dfrac{x^2+3x+2}{x^2+5x+4} = \dfrac{(x+2)(x+1)}{(x+4)(x+1)}$

$= \dfrac{x+2}{x+4}$

i $\dfrac{x^2+x-12}{x^2-9x+18} = \dfrac{(x+4)(x-3)}{(x-6)(x-3)}$

$= \dfrac{x+4}{x-6}$

Answers Exam Set 1

3 $6x^3 + 3x^2 - 84x = 3x(2x^2 + x - 28)$

$= 3x(2x - 7)(x + 4)$

$6x^2 - 33x + 42 = 3(2x^2 - 11x + 14)$

$= 3(x - 2)(2x - 7)$

$$\frac{6x^3 + 3x^2 - 84x}{6x^2 - 33x + 42} = \frac{3x(2x - 7)(x + 4)}{3(x - 2)(2x - 7)}$$

$$= \frac{x(x + 4)}{(x - 2)}$$

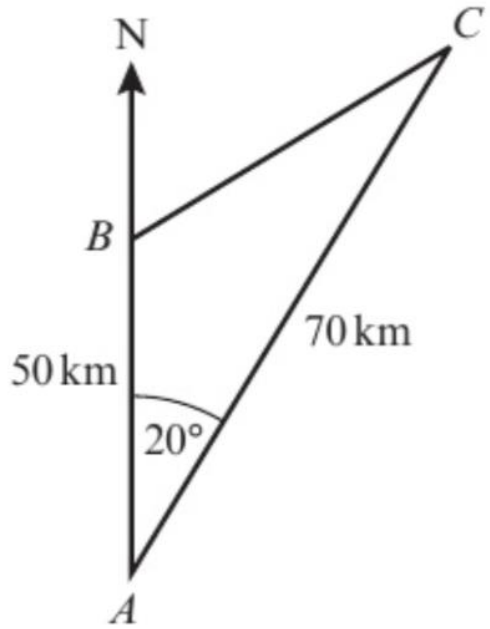
$a = 1, b = 4, c = -2$

Exam Questions 2

The diagram shows three cargo ships, A, B and C, which are in the same horizontal plane. Ship B is 50 km due north of ship A and ship C is 70 km from ship A. The bearing of C from A is 020°.

a Calculate the distance between ships B and C, in kilometres to 3 s.f. **(3 marks)**

b Calculate the bearing of ship C from ship B. **(4 marks)**



a $a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 70^2 + 50^2 - 2 \times 70 \times 50\cos 20°$

$a^2 = 4900 + 2500 - 6577.848...$

$a^2 = 822.151\,65...$

So $a = 28.673...$

So the distance between ships B and C is 28.7 km (3 s.f.).

b $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$\cos B = \dfrac{28.673^2 + 50^2 - 70^2}{2(28.673)(50)}$

$\cos B = \dfrac{822.151\,65 + 2500 - 4900}{2867.3187}$

$\cos B = -0.550\,28...$

$B = \cos^{-1} 0.550\,28...$

$= 123.3867...°$

The bearing is $180° - 123.3867° = 56.6°$.

So the bearing of ship C from ship B is 056.6°.

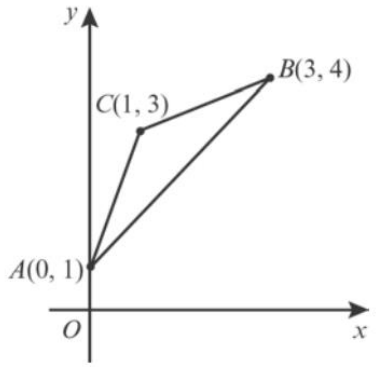
Exam Questions 2

The three points A, B and C, with coordinates $A(0, 1)$, $B(3, 4)$ and $C(1, 3)$ respectively, are joined to form a triangle.

a Show that $\cos \angle ACB = -\frac{4}{5}$ **(5 marks)**

b Calculate the area of $\triangle ABC$. **(2 marks)**

a)



a Use Pythagoras' theorem.

$$AC = \sqrt{(1-0)^2 + (3-1)^2}$$
$$= \sqrt{5}$$
$$= b$$

$$BC = \sqrt{(3-1)^2 + (4-3)^2}$$
$$= \sqrt{5}$$
$$= a$$

$$AB = \sqrt{(3-0)^2 + (4-1)^2}$$
$$= \sqrt{18}$$
$$= c$$

Using the cosine rule

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos C = \frac{5 + 5 - 18}{2 \times \sqrt{5} \times \sqrt{5}}$$

$$= \frac{-8}{10}$$

$$= \frac{-4}{5}$$

b)

Using the area formula:

area of $\Delta ABC = \frac{1}{2}ab\sin C$

$$= \frac{1}{2} \times \sqrt{5} \times \sqrt{5} \times \sin C$$

$$= 1.5 \text{ cm}^2$$

GCSE style questions

1. (*a*) Find the value of $3x^3 + 2ax^2 - 4x + 5a$ when $x = -3$.

(2)

(*b*) Find the value of a when $69 + 23a = 0$.

(1)

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2. Three Bags, *A*, *B* and *C*, each contain 1 red marble and some green marbles.

Bag *A* contains 1 red marble and 9 green marbles only
Bag *B* contains 1 red marble and 4 green marbles only
Bag *C* contains 1 red marble and 2 green marbles only

Sasha selects at random one marble from Bag *A*.
If he selects a red marble, he stops selecting.
If the marble is green, he continues by selecting at random one marble from Bag *B*.
If he selects a red marble, he stops selecting.
If the marble is green, he continues by selecting at random one marble from Bag *C*.

(*a*) Draw a tree diagram to represent this information.

(2)

(*b*) Find the probability that Sasha selects 3 green marbles.

(2)

3. (*a*) Rearrange the equation $1 - \frac{x^2}{2} - 2x - \frac{1}{2} = 0$ into the form $ax^2 + bx + c = 0$.

(1)

(*b*) Solve the equation found in part (*a*).

(1)

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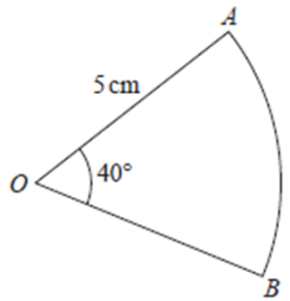
4. Show that $\frac{(x+1)^2 \times (10x+10) - (5x^2+10x) \times 2(x+1)}{(x+1)^4} = \frac{A}{(x+1)^n}$ where A and n are integers to be found.

(2)

5. Find the area of the sector AOB.



(2)

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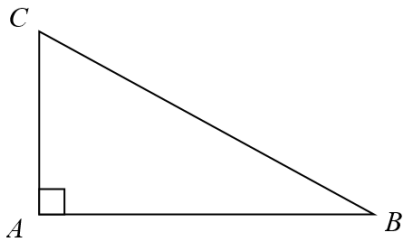
6. (a) Find x when $\frac{4-3x}{1+2x} = -\frac{4}{3}$

(2)

(b)



The diagram shows a right-angled triangle ABC where $AB = x^2 - x$ and $AC = \frac{3}{2}x^2 - 4x$.

Find the distance BC when $x = 4$.

(2)

7. (*a*) Write f(x) = $2x^2 + 4x + 9$ in the form $a(x + b)^2 + c$.

(3)

(*b*) Sketch the curve with equation $y = 2x^2 + 4x + 9$, showing any points of intersection with the coordinate axis and the coordinates of any turning point.

(3)

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8. Find x when $10 (\cos x)^2 = 9$, $0° < x < 90°$.

(2)

9. Magali is studying the mean total cloud cover, in oktas, using data from the large data set. The daily mean total cloud cover for all 184 days from the large data set is summarised in the table below.

Daily mean total cloud cover (oktas)	0	1	2	3	4	5	6	7	8
Frequency (number of days)	0	1	4	7	10	30	52	52	28

One of the 184 days is selected at random.

(*a*) Find the probability that it has a daily mean total cloud cover of 6 or greater.

(1)

There were 28 days that had a daily mean total cloud cover of 8. For these 28 days the daily mean total cloud cover for the **following** day is shown in the table below.

Daily mean total cloud cover (oktas)	0	1	2	3	4	5	6	7	8
Frequency (number of days)	0	0	1	1	2	1	5	9	9

(*b*) Find the proportion of these days when the daily mean total cloud cover was 6 or greater.

(1)

10. (*a*) Solve the simultaneous equations

$$x + 880y = 1100$$
$$x + 300y = 680$$

(1)

(*b*) Find the least value of n when $2n - (428 + 0.84n) > 0$

(1)

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11. (*a*) Expand and simplify $y = x(x + 2)(x - 4)$.

(1)

(*b*) Find the value of $\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2$ when $x = 2$.

(1)

(*c*) Expand and simplify $y = (x + 2)^2(3x^2 - 20x + 20)$.

(2)

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12. Given that $a - b = \frac{a}{b}$, show that $a = \frac{b^2}{b-1}$.

(2)

13. Work out how far a car moving at 60 km h^{-1} travels in 0.8 seconds, giving your answer in metres.

(1)

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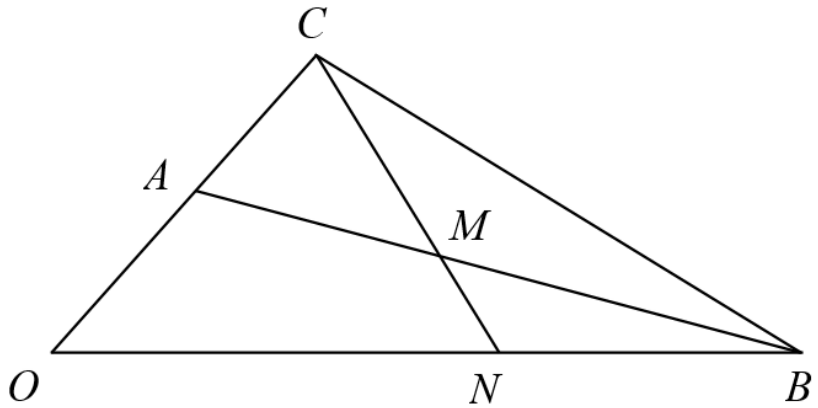
14. If n is an integer greater than 1, show, by considering both odd and even numbers, that $n^2 + 2$ is not divisible by 4.

(4)

15.



The diagram shows a sketch of triangle OAB.

The point C is such that $\overrightarrow{OC} = 2\overrightarrow{OA}$.

The point M is the midpoint of AB.

The straight line through C and M cuts OB at the point N.

Given $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$, **f**ind $\overrightarrow{CM}$ in terms of **a** and **b**.

(2)

16. Use the iteration formula

$$x_{n+1} = 2x_n^{1-x_n}$$

with $x_1 = 1.5$ to find x_4 to 3 decimal places.

(2)

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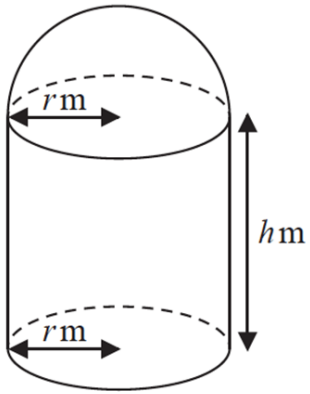
17. (*a*) A runner finishes a race in $24 + (6 \times 1.05) + (6 \times 1.05^2)$ minutes. Find this time in hours, minutes and seconds.

(1)

(*b*) A runner finishes a race in $24 + 6.3 \times \dfrac{(1.05^{16} - 1)}{1.05 - 1}$ minutes. Find this time in hours, minutes and seconds.

(2)

19.



[A sphere of radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$]

A manufacturer produces a storage tank modelled in the shape of a hollow circular cylinder closed at one end with a hemispherical shell at the other end as shown in the diagram above.

The cylinder has radius r metres and height h metres and the hemisphere has radius r metres. The volume of the tank is 6 m^3.

Show that the surface area of the tank, in m^2, is given by

$$\frac{12}{r}+\frac{5}{3}\pi r^2$$

(4)